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## INVERSE SCATTERING FOR SCATTERING DATA WITH POOR REGULARITY OR SLOW DECAY

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**1. Introduction.** Motivation to study the inverse scattering problem for scattering data with poor regularity or slow decay is an application which will be given in two subsequent papers [7, 21] for the Cauchy problem of the Korteweg-deVries equation ( $KdV$ )  $u_t - 6uu_x + u_{xxx} = 0$  with irregular initial profile as, e.g., a smooth enough box shaped potential or a steplike a smoothed Heavyside function [4,5].

If we consider  $u(x)$  as a potential for the Schrödinger equation  $-y''(x) + u(x)y(x) = k^2y(x)$  we can associate to  $u$ , by a well known procedure [8,9], the scattering data of which a part is given by the so called scattering matrix  $(T_+, R_+, T_-, R_-)$ . To find a solution  $u(x, t)$  of the  $KdV$  ( $t > 0$ ) it is enough to study the evolution of the scattering in time and to construct  $u(x, t)$  by the inverse problem [3, 4, 5, 7, 10, 11, 12, 13]. Often, however, the evolution of the scattering data, especially  $R_-$ , does not stay within the set where the inverse problem was known to be solvable [4, 5].

Let us briefly outline the organization of the paper. In §2 we discuss the Marchenko equation in  $L_2(R_-)$ . In §3 we study the inverse scattering problem under weaker decay and regularity properties of  $R_-$  and its Fourier transform than in [8, 9].

Let us introduce the following notation. Let  $f$  be a complex valued function defined on  $\mathbf{R}$ . By  $\tau_x f$  we denote the translated function  $\tau_x f(y) := f(x + y)$  ( $x$  and  $y$  in  $\mathbf{R}$ ). If  $h \neq 0$  we denote by  $\Delta_h f$  the differential quotient  $(\Delta_h f)(x) := \frac{f(x+h)-f(x)}{h}$ .

Let  $f$  be in  $L_2(\mathbf{R})$ . By  $\hat{f}$  we denote the Fourier transform  $\hat{f}(k) := \int_{-\infty}^{\infty} f(x)e^{2ikx}dx$ . By  $\tau_x f$  we define the operator on  $L_2(\mathbf{R}_-)$  defined by

$$\tau_x f(g)(y) := \int_{-\infty}^0 \tau_z f(x+z)g(z)dz \quad (g \text{ in } L_2(\mathbf{R}_-)).$$

By  $'$  or  $\partial_x$  we denote the derivation with respect to  $x$ . For a complex